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A GENERALIZED RADAR OUTPUT SIMULATION

J. F. A. Ormsby
S. H. Bickel

JULY 1969

Prepared for

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 4963
Prepared by
THE MITRE CORPORATION
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FOREWORD

The work reported in this document was performed by The MITRE Corporation, Bedford, Massachusetts, for Electronic Systems Division of the Air Force Systems Command under Contract AF 19(628)-2390. This information was originally published as Working Paper W-7346, The MITRE Corporation, 19 October 1964.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

ANTHONY P. TRUNFIO, Technical Advisor
Development Engineering Division
Directorate of Planning and Technology

ABSTRACT

Using the outputs from the simulation of overall body motion related to observations at a complex of earth stations, this report develops a model for generating the scattering matrix and radar output voltages.

The general multistatic case is treated using a geometry which simplifies the data gathering and merges conveniently with the motion simulation. The monostatic case is given separate treatment due to certain simplifications possible in this situation.

Variations to account for polarization modes, Faraday rotation and noise are also included.

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1.0 INTRODUCTION

Simulation of the relationship between the overall motion of a satellite and observations at a complex of earth stations has been previously discussed. This motion simulation specifies the orientation of a body fixed axes system $\{b'\}$ to the radar operating system for site i, $\{r_i\}$ in terms of three Eulerian angles $\beta_{1i}(t)$, $\beta_{2i}(t)$, $\beta_{3i}(t)$.†

The present report is concerned with using the β angles in conjunction with scattering matrix data. The simulation develops outputs from the scattering data which account for polarization modes, Faraday rotation, and noise.

The general multistatic case is treated with each site acting as a transmitter or receiver or both. This is equivalent for simulation to considering any two sites as a transmitter - receiver pair, that is the bistatic case. The monostatic case for one or more sites where each radar receives the echo from its own transmission only, is a specialization of the bistatic case and allows for certain simplifications in the simulation.

The scattering data can be stored and is obtained either from measurement or calculation. To account for either of these possibilities, the expression data gathering will be used. The operating model for both the bistatic and monostatic cases provides simplicity and compatibility of the data gathering and motion outputs.

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†See Reference 1. (Earlier more detailed accounts by this author exist as internal reports only.)

The output voltages can be related to the entries of the scattering matrix. The time patterns of the scattering matrices in various orthogonal polarization modes can then be correlated with specific body motion to aid in developing signature analysis techniques for describing body configuration, orientation, and motion.

2.0 RADAR SIMULATION MODEL

The basic operation in the simulation merges the scattering data appropriate to the orientation corresponding to the motion. This involves developing a parameter set which uniquely specifies the scattering data for a given orientation. The set must (1) be available from the motion simulation and (2) allow for a convenient scheme of measurement or calculation.

The model developed for the multistatic (bistatic) case accomplishes these goals. Due to certain simplifications, however, the monostatic case will be discussed first. Details on the scattering matrix and output voltages will be given in the next section.

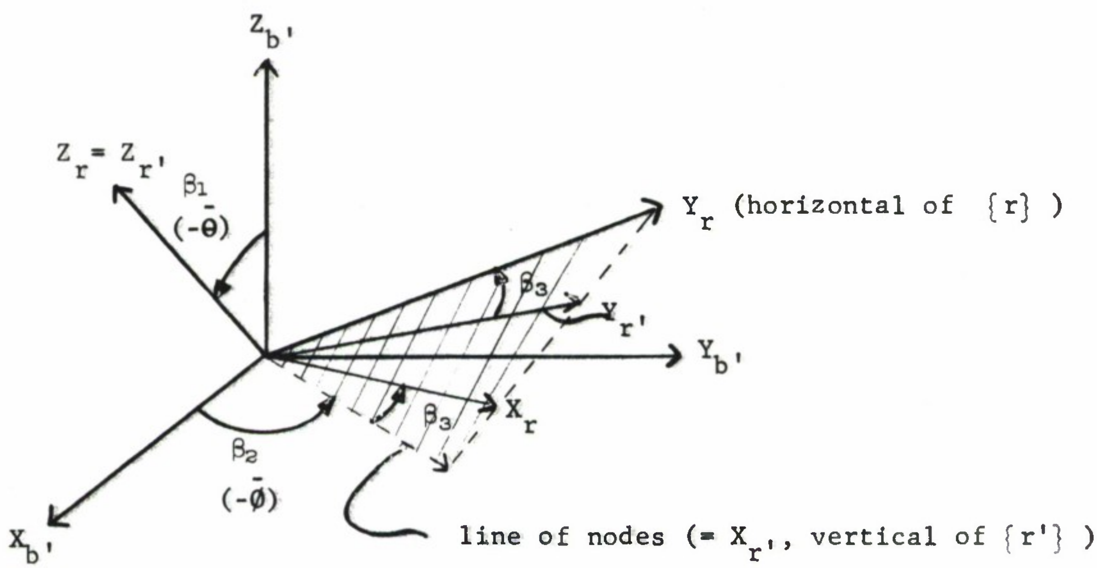
2.1 Monostatic Case

With n monostatic sites, the simulation is an n-fold repetition of that for a single site except of course for calculations which depend on site location as specified by longitude and latitude angles (λ_i, η_i) for site i.[†] The net effect will be a possible n-fold increase in processing with no changes in the data

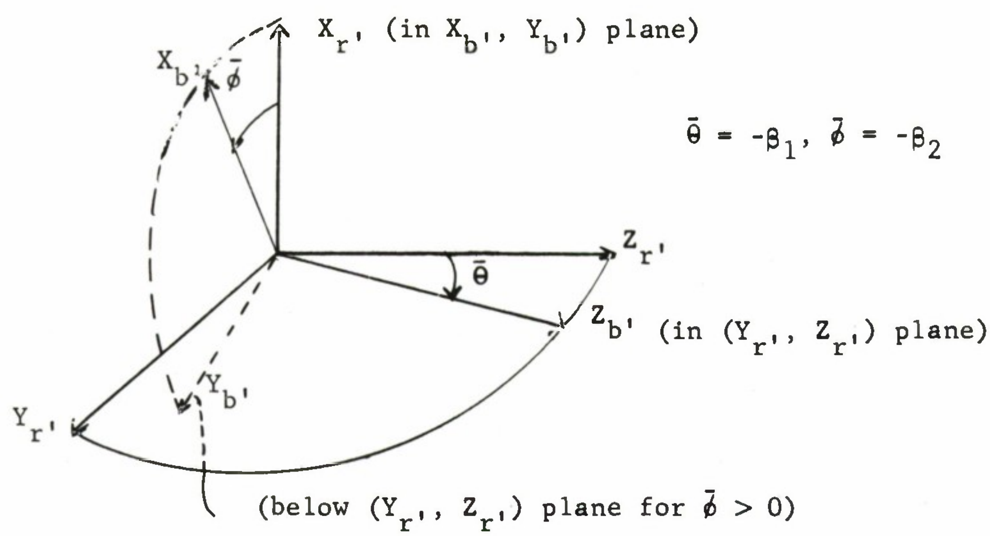
[†] See Reference 1. (Earlier more detailed accounts by this author exist as internal reports).

gathering requirement. It is thus sufficient to consider a single site acting as a transmitter and receiver.

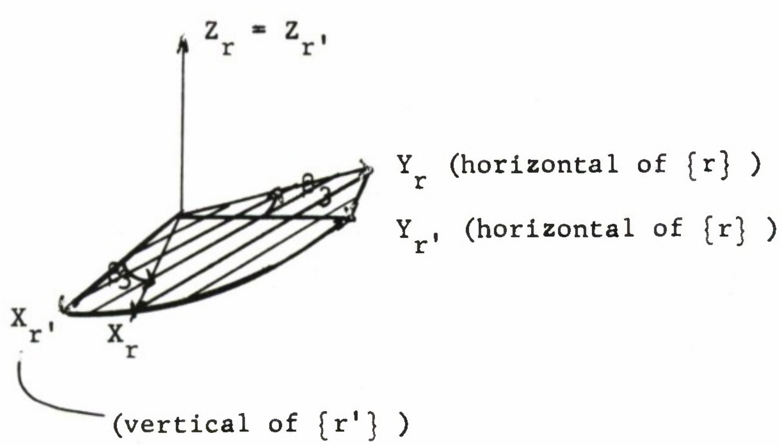
With $\{b'\}$ the geometrical body axes system and $\{r\}$ the radar operating system, their relationship can be described by the three Eulerian angles $\beta_1(t)$, $\beta_2(t)$, $\beta_3(t)$. The determination of these angles in terms of orbital and local motion of the satellite is discussed in references 1 and 2. The geometry can be shown as follows where the $\{r'\}$ system (see below) is included for completeness.



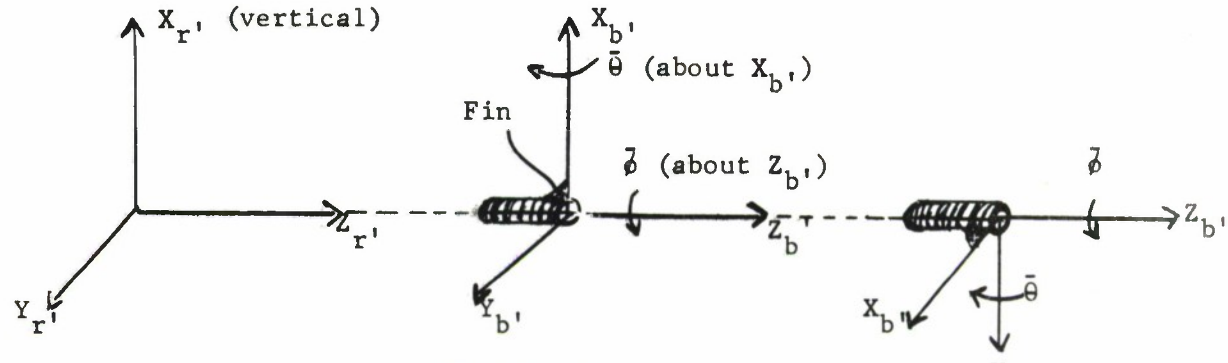
The direction Z_r is the radar line of sight and so β_3 represents rotation in a plane perpendicular to the line of sight. Hence β_3 is a polarization angle and can be accounted for by rotation of the scattering matrix as shown in the next section. Thus only the aspect angles β_1 and β_2 are required. A convenient choice for β_3 is zero and is so chosen for the data gathering setup where $\{r'\}$ is the radar system with $\beta_3 = 0$ or equivalently the radar measurement system. It is noted that the $\{r'\}$ system is fixed relative to $\{b'\}$.



The relation of $\{r\}$ to $\{r'\}$ is simply the rotation β_3 .



The physical orientation of the body in the data gathering setup can also be illustrated as follows:



(i) $\bar{\theta} = \bar{\phi} = 0$

(ii) $\bar{\theta} = 0$, $\bar{\phi} = \pi/2$

The physical movement of the body consists of first a rotation about $X_{b'}$ by $\bar{\theta}$ followed by a rotation about $Z_{b'}$ of $\bar{\phi}$. In relating the $\{r'\}$ system to the $\{b'\}$ system displaced by $\bar{\theta}$ and $\bar{\phi}$ we write

$$T_{r',b'} = \begin{pmatrix} \cos\bar{\phi} & \sin\bar{\phi} & 0 \\ -\sin\bar{\phi} & \cos\bar{\phi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\bar{\theta} & \sin\bar{\theta} \\ 0 & -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}$$

The $\bar{\theta}$ rotation about $X_{r'}$ takes $Z_{r'}$ into $Z_{b'}$ and the $\bar{\phi}$ rotation about $Z_{r'}$ takes $X_{r'}$, $Y_{r'}$ into $X_{b'}$, $Y_{b'}$. Throughout $Z_{b'}$ remains in the $(Y_{r'}, Y_{b'})$ plane and $X_{r'}$ remains in the $(X_{b'}, Y_{b'})$ plane.

The relation of $\{r\}$ to $\{b'\}$ is given by a rotation of $-\beta_3$ about Z_r, followed by a rotation of $-\beta_1 = \bar{\theta}$ about line of nodes, $X_{r'}$, ($= X_r$ after $-\beta_3$ rotation), followed by a rotation of $-\beta_2 = \phi$ about $Z_{b'}$ ($= Z_r$ after $-\beta_1$ rotation).

$$T_{r,b'} = \underbrace{\begin{pmatrix} \cos\beta_2 & -\sin\beta_2 & 0 \\ \sin\beta_2 & \cos\beta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta_1 & -\sin\beta_1 \\ 0 & \sin\beta_1 & \cos\beta_1 \end{pmatrix}}_{T_{r',b'}}\underbrace{\begin{pmatrix} \cos\beta_3 & -\sin\beta_3 & 0 \\ \sin\beta_3 & \cos\beta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{T_{r,r'}}$$

$$= T_{r',b'}T_{r,r'}$$

The data is obtained for N_θ values of $\bar{\theta}$ with $\Delta\bar{\theta} = \pi/N_\theta$ and N_ϕ values of $\bar{\phi}$ with $\Delta\bar{\phi} = 2\pi/N_\phi$. For unperturbed bodies of revolution $N_\phi = 1$. The data consists of the magnitude and phase of the four entries of the scattering matrix (see next section) for a given $(\bar{\theta}, \bar{\phi})$ pair. Accounting for magnitude and phase by complex entries σ, the format of the data will be

$$[\bar{\theta}_i, \bar{\phi}_j; \sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}]; \quad 1 \le i \le N_\theta$$
$$1 \le j \le N_\phi$$

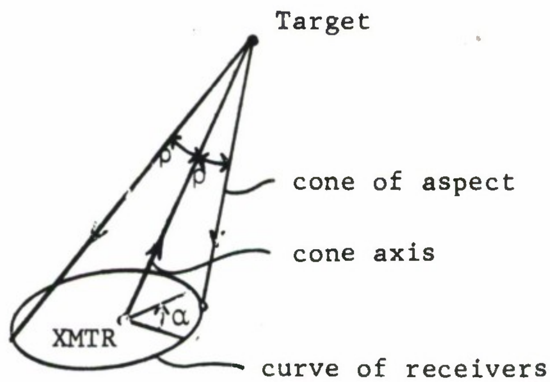
From the motion simulation relating $\{r\}$ and $\{b'\}$, the angles β_1, β_2, and β_3 are obtained. With $\bar{\theta} = -\beta_1$, $\bar{\phi} = -\beta_2$ the proper σ values are found. For values of $\bar{\theta}$ and $\bar{\phi}$ which are intermediate to the $\bar{\theta}_i$, $\bar{\phi}_j$ of the data, one or two dimensional interpolation on the σ values using general interpolation formulae is applied (see Reference 2).

Normalization can be included so that the maximum magnitude of the σ values is one. Also for convenience in using the radar wave equation to include variation with range, the σ can be in units of cross section.

The angle β_3 is used to rotate the scattering matrix from $\{r'\}$ to $\{r\}$.

2.2 Multistatic (Bistatic) Case

In this case, we deal with different transmitter and receiver sites. Basically the aspect of each line of sight to the target must be taken into account. The aspect angle (bistatic angle) between the lines of sight is not sufficient. This is easily seen by considering the following example.



Receivers on the curve of receivers all have the same bistatic angle ρ to the transmitter but have different aspects to the body due to different α values.

Thus, we should expect the extension of the data parameter set from two to four values in order to provide similar accuracy. Equivalently for any pair of sites i and j and each $(\bar{\theta}, \bar{\phi})_i$ pair from site i to the target, we can consider as many $(\bar{\theta}, \bar{\phi})_j$ pairs from site j to the target as there are $(\bar{\theta}, \bar{\phi})_i$ pairs in the monostatic case. In other words, if there are n monostatic $(\bar{\theta}\ \bar{\phi})$ values there should be n^2 bistatic data values.

We see that now the body has to be related to two sites rather than one. In the monostatic case it was convenient to take the $\{r\}$ or $\{r'\}$ system itself as a reference when relating the $\{b'\}$ system to this one site. If in the bistatic case we choose either of the sites on which to reference the body movement, then relating the body to the other site becomes inconvenient.

To avoid these problems a separate pedestal system $\{p\}$ uniquely defined by the two radar systems $\{r_1\}$ and $\{r_2\}$ or $\{r_1'\}$ and $\{r_2'\}$ is used to again provide a single system to which the body motion can be referred. In the monostatic case the $\{r\}$ or $\{r'\}$ systems take the place of $\{p\}$. In terms of the operating systems $\{r_1\}$ and $\{r_2\}$, we define the $\{p\}$ system as follows:†

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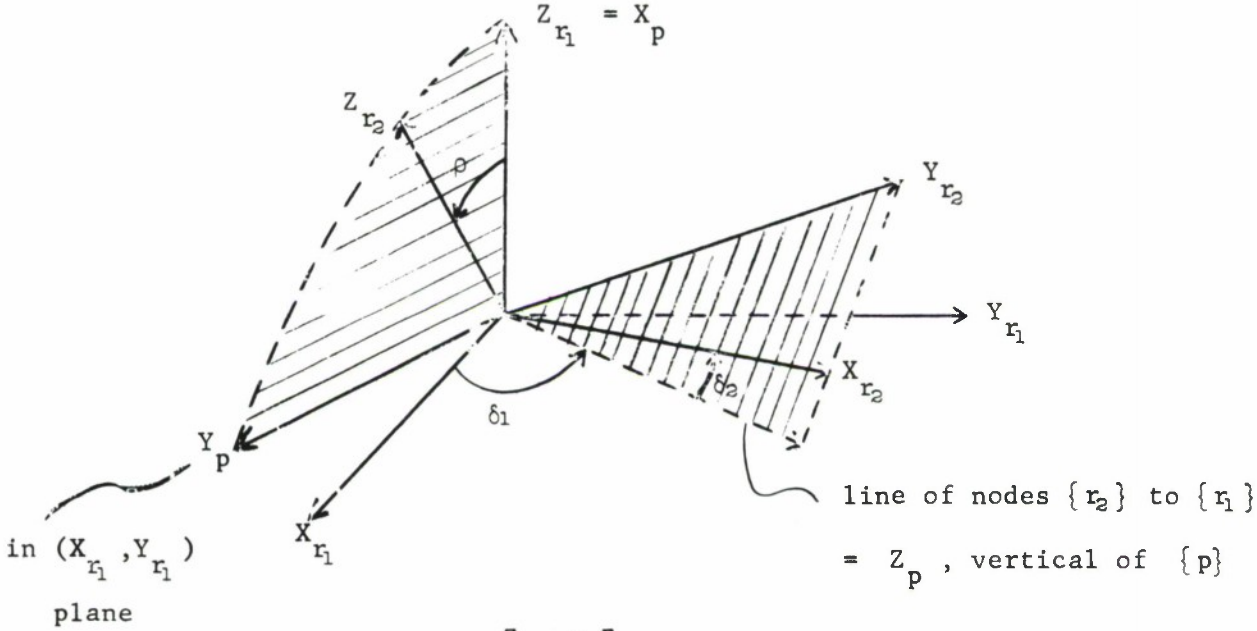
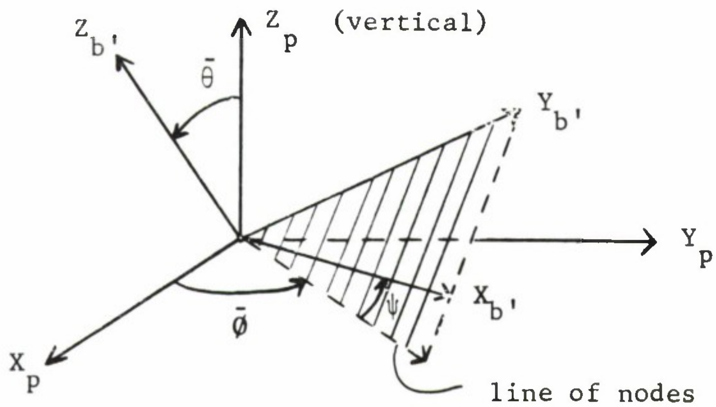
† Although other choices are possible the choice of X_p in the Z_{r_1} direction is particularly convenient.



Unit Vectors: $Z_p = \dfrac{Z_{r_1} \times Z_{r_2}}{\sin \rho}$, $X_p = Z_{r_1}$, $Y_p = Z_p \times X_p$

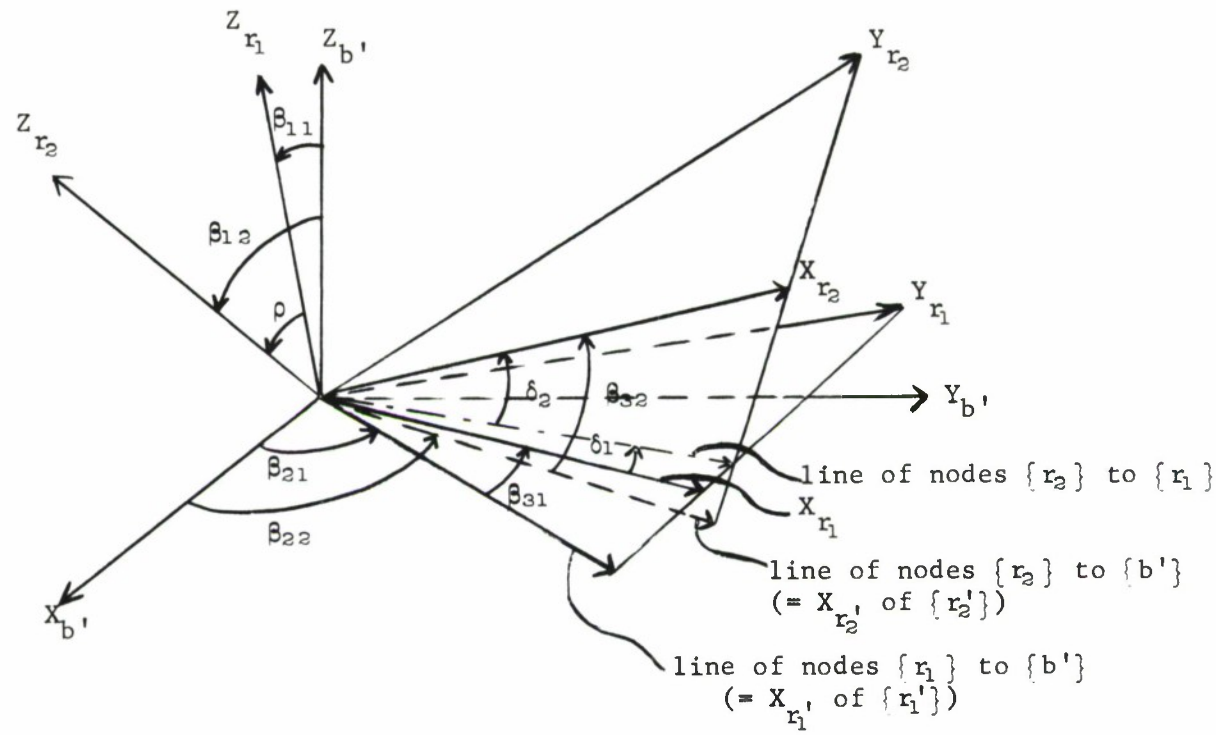
Note the vertical of the $\{p\}$ system, Z_p, is given by the line of nodes formed by the (X,Y) planes in the $\{r_1\}$ and $\{r_2\}$ systems rather than the vertical of the $\{r'\}$ system given by the line of nodes between the $\{r\}$ and $\{b'\}$ systems in the monostatic case. As in the monostatic case $\{r_1\}$ and $\{r_1'\}$ are related by the same kind of $T_{r,r'}$ in terms of the polarization angle β_{31} and $\{r_2\}$ and $\{r_2'\}$ by the polarization angle β_{32}. The line of sight directions are again given by Z_{r_1} and Z_{r_2} .

The $\{p\}$ system is considered fixed relative to $\{b'\}$ as shown.



Whereas in the monostatic case $\{r\}$ to $\{b'\}$ was specified by $\beta_1 (= -\bar{\theta})$, $\beta_2 (= -\bar{\emptyset})$, and β_3 to obtain the scattering matrix from scattering data based on $\{r'\}$ related to $\{b'\}$ by $\bar{\theta}$ and $\bar{\emptyset}$, now in the bistatic case, $\{r_1\}$ to $\{b'\}$ and $\{r_2\}$ to $\{b'\}$ relate $\{r_1\}$ to $\{r_2\}$ and these relations allow a specification of $\{p\}$ from Z_{r_1} and Z_{r_2} and hence of $\{p\}$ to $\{b'\}$. The three Eulerian angles $\bar{\theta}$, $\bar{\emptyset}$, and $\bar{\psi}$ relating $\{p\}$ to $\{b'\}$ provide three of the four entry parameters for looking up the scattering data obtained by measurement or calculation. The fourth parameter is the bistatic angle ρ between Z_{r_1} and Z_{r_2} obtained by relating $\{r_1\}$ to $\{r_2\}$.

The geometry can be indicated by the drawing below.



β_{11} , β_{21} , β_{31} from $T_{r,b'}$

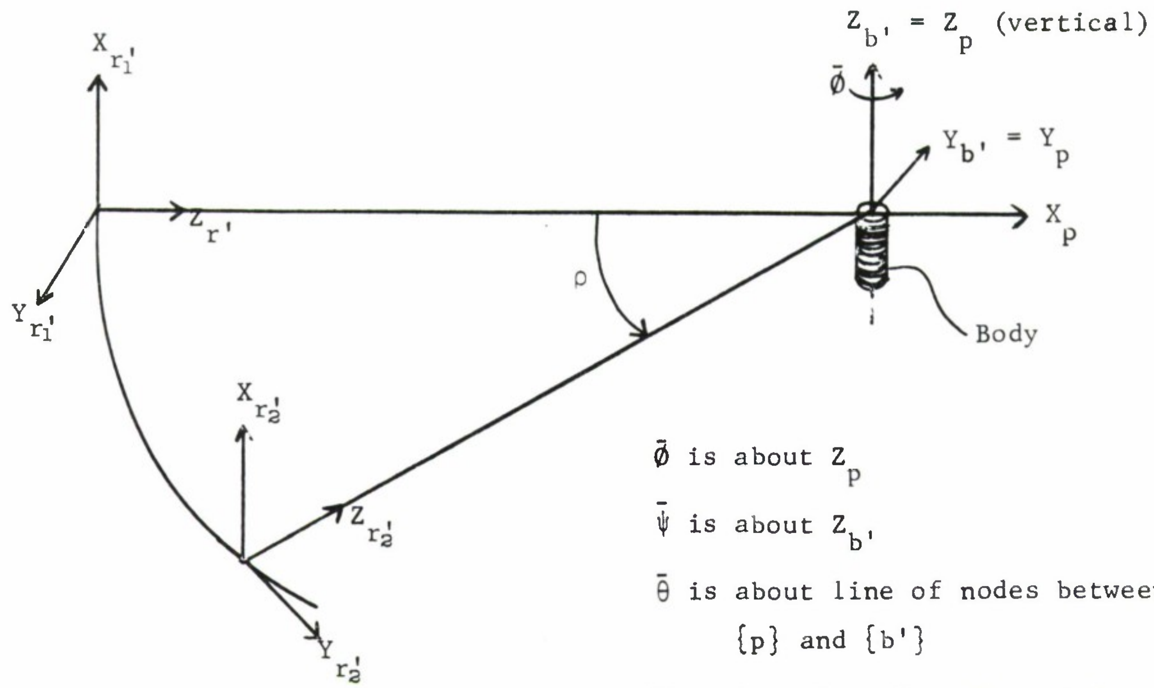
β_{12} , β_{22} , β_{32} from $T_{r_2,b'}$

ρ , δ_1 , δ_2 from $T_{r_1,r_2} = (T_{r_2,b'})^{-1} \, T_{r_1,b'}$

With six degrees of freedom indicated by the β angles and with β_{31} and β_{32} the polarization angles between $\{r_1\}$ and $\{r_1'\}$ and $\{r_2\}$ and $\{r_2'\}$ respectively, the bistatic case reduces to a four entry look up implied by the four degrees of freedom β_{11} , β_{12} , β_{21} , β_{22} .

The single aspect angle ρ simplifies the data gathering setup by allowing the $\{r_1'\}$ and $\{r_2'\}$ systems to have relative motion in a plane (horizontal).

The physical orientation of the body in the data gathering setup can be viewed as follows where the $\{p\}$ and $\{b'\}$ systems are aligned that is with $\bar{\theta} = \bar{\phi} = \bar{\psi} = 0$.



$\bar{\phi}$ is about Z_p

$\bar{\psi}$ is about $Z_{b'}$

$\bar{\theta}$ is about line of nodes between $\{p\}$ and $\{b'\}$

Y_{r_1}, Z_{r_1}, Y_{r_2}, Z_{r_2}, X_p and Y_p remain in one plane (horizontal)

as before, $Z_p = \dfrac{Z_{r_1'} \times Z_{r_2'}}{\sin \rho}$, $X_p = Z_{r_1'}$, $Y_p = Z_p \times X_p$.

With rotation of $\{b'\}$ about Z_p given by $\bar{\phi}$ it is convenient in a measurement setup to choose Z_p as the vertical.

The data is obtained for N_θ, N_ϕ, N_ψ, and N_ρ values of $\bar{\theta}$, $\bar{\phi}$, $\bar{\psi}$, and ρ respectively. For unperturbed bodies of revolution $N_\psi = 1$. Also N_ρ is limited since $\rho_{MAX} < 180^\circ$. For distant targets there can result $\rho_{MAX} < 90^{\circ}$. The data format now becomes

$$[\bar{\theta}_i, \bar{\phi}_j, \bar{\psi}_k, \rho_s; \; \sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}]; \quad \begin{array}{l} 1 \le i \le N_\theta \\ 1 \le j \le N_\phi \\ 1 \le k \le N_\psi \\ 1 \le s \le N_\rho \end{array}$$

Thus, the outputs from the motion simulation relating $\{r_1\}$ to $\{b'\}$ and $\{r_2\}$ to $\{b'\}$ provide β_{11}, β_{21}, β_{31}, β_{12}, β_{22}, and β_{32}. From these $\{p\}$ and ρ can be found and so $\{p\}$ to $\{b'\}$ for $\bar{\theta}$, $\bar{\phi}$, and $\bar{\psi}$ providing the four entry parameter set to obtain the σ values. Finally β_{31} and β_{32} allow for the proper rotation of the scattering matrix. (See next section.)

The comments on normalization and interpolation in the monostatic case carry over to the bistatic case.

3.0 SCATTERING MATRIX AND OUTPUTS

3.1 Scattering Matrix

The scattering matrix relates the complex incident wave on a scattering body to the reflected wave. Expressed in the left circular-right circular coordinate system we express the scattering matrix, S, having complex entries as

$$S = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

where, for example, $\sigma_{11} = \dfrac{\text{Left circular reflected (receive) mode}}{\text{Left circular incident (transmit) mode}}$

The meaning of all entries can be given as follows where L = left circular and R = right circular.

Entry	Transmit Mode	Receive Mode
σ_{11}	L	L
σ_{12}	R	L
σ_{21}	L	R
σ_{22}	R	R

The left circular mode rotates in a clockwise direction. For transmission in the +Z direction and a positive rotation, β, in the (X,Y) plane taken counter clockwise†, then the rotation matrix in the left circular - right circular coordinate system (in the (X,Y) plane) is

$$R = \begin{pmatrix} e^{-i\beta} & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

since β acts as $-\beta$ on the left circular and β on the right circular.

The direction of propagation reverses between transmission and reception so that

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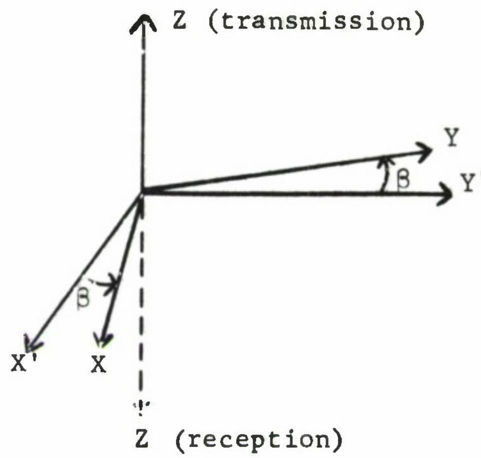
† For a right handed coordinate system.

β (transmission) = $-\beta$ (reception)



The effect of a rotation, $R(\beta)$, transforms the scattering matrix S to S' by

$$S' = R^{-1}(\beta') \, S \, R(\beta)$$

In our case since β (transmission) = β' (reception)

$$R^{-1}(\beta') \text{ becomes } R^{-1}(-\beta) = R(\beta)$$

so that

$$S' = R(\beta) \, S \, R(\beta)$$

The rotation of the X_r, Y_r axes in the $\{r\}$ system from the $X_{r'}$, $Y_{r'}$ axes in the $\{r'\}$ system was given by the polarization aspect angle β_3 , see previous section. The effect of rotation β_3 on S becomes

$$S' = \begin{pmatrix} \sigma_{11} e^{-j2\beta_3} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} e^{j2\beta_3} \end{pmatrix}$$

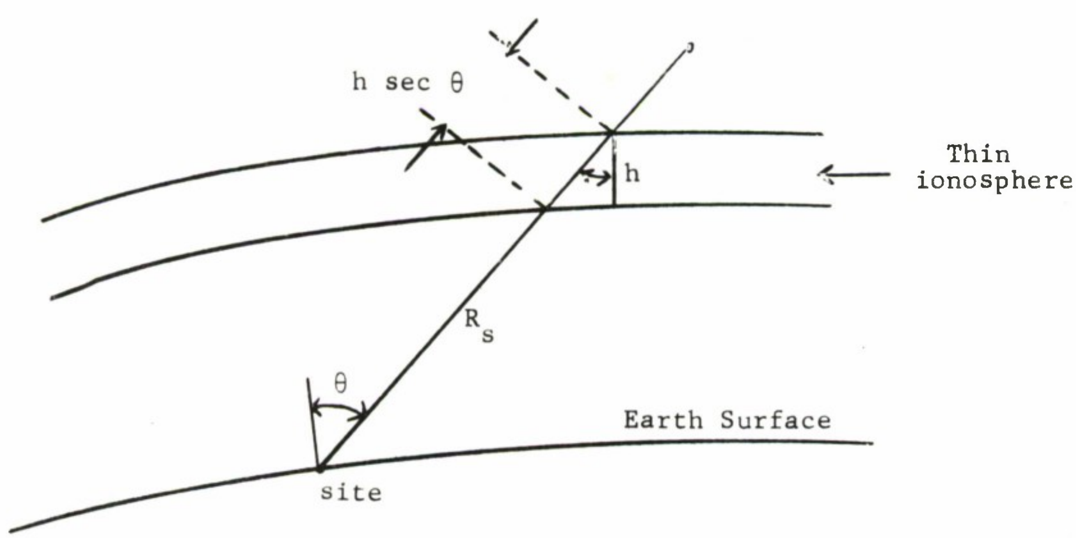
The values of the entries σ_{11}, σ_{12}, σ_{21}, σ_{22} are obtained either from scattering measurements made by the measurement radar on various body shapes or by calculation.

In the bistatic case with site 1 transmitter and site 2 receiver we have

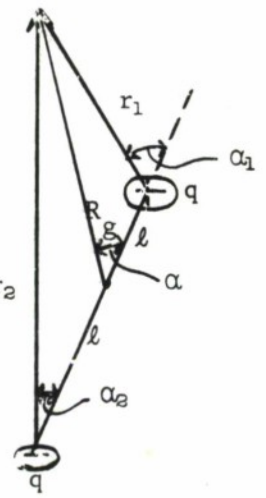
$$S' = R(\beta_{32})\ S\ R\ (\beta_{31})$$

3.2 Faraday Rotation

To account for Faraday rotation a simple model which considers the earth as a pair of magnetic monopoles is used together with a thin ionosphere for magnetic variation.



Slant Range Through the Ionosphere



q = magnetic charge.

ℓ = magnetic radius.

Earth as a Pair of Magnetic Monopoles

Magnetic potential $= V = q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

For $R_g >> \ell$, $r_1 \simeq r_2 \cong R_g$ and

$$\alpha_1 \cong \alpha_2 \cong \alpha$$

Then $V \simeq 2q\rho_1 \frac{\cos\alpha}{R_q^2}$ = dipole approximation of the multipole expansion

with $2q\rho_1$ the magnetic moment.

Next the magnetic field is given by

$$i = \nabla V$$

and is related to the site through R_g by $r_1 = R_g - \rho_1$, $r_2 = R_g + \rho_1$,

and a transformation from geocentric to topocentric coordinates by the transformation $T_{g,s}$ (see footnote on page 1).

Then the two way Faraday rotation, $2\theta_F$, is given by

$$2\theta_F = \frac{C\rho(h)}{f^2} (\bar{i} \cdot \hat{R}_s) \sec \theta$$

where C is a constant

$\rho(h)$ is the integrated electron density across the ionosphere for a unit base cross-section.

f is the transmission frequency.

R_s is the line of sight distance from the site to the target.

θ is the elevation angle at the site from the earth normal to the line of sight.

$(\cdot)$ indicates vector dot product.

The effect of the Faraday rotation θ_F on the scattering matrix, S , is

$$S' = R^{-1}(\theta_F)\ S\ R(\theta_F)$$

where $R(\theta_F)$ is the rotation transformation for transmission and $R^{-1}(\theta_F) = R(-\theta_F)$.

Whereas for polarization aspect the rotation angle β , reverses in sign between transmission and reception giving

$$S' = R(\beta)\ S\ R(\beta)$$

For Faraday rotation no reversal occurs so that

$$S' = R(-\theta_F)\ S\ R(\theta_F)$$

giving $$S' = \begin{pmatrix} \sigma_{11} & \sigma_{12}\, e^{+j2\theta_F} \\ \sigma_{21}\, e^{-j2\theta_F} & \sigma_{22} \end{pmatrix}$$

For a polarization rotation of β_3 and two-way Faraday rotation, $2\theta_F$

$$
S' = \begin{pmatrix} \sigma_{11} e^{-i2\beta_3} & \sigma_{12} e^{j2\theta_F} \\ \sigma_{21} e^{-j2\theta_F} & \sigma_{22} e^{j2\beta_3} \end{pmatrix}
$$

In the bistatic case both θ_F and β_3 can differ on transmission and reception. We have β_{31}, β_{32}, θ_T and θ_{F_2} so that S' is given by

$$
S' = R(\beta_{32} - \theta_{F_2})\ S\ R\ (\beta_{31} + \theta_{F_1})
$$

where the rotation matrix is given by

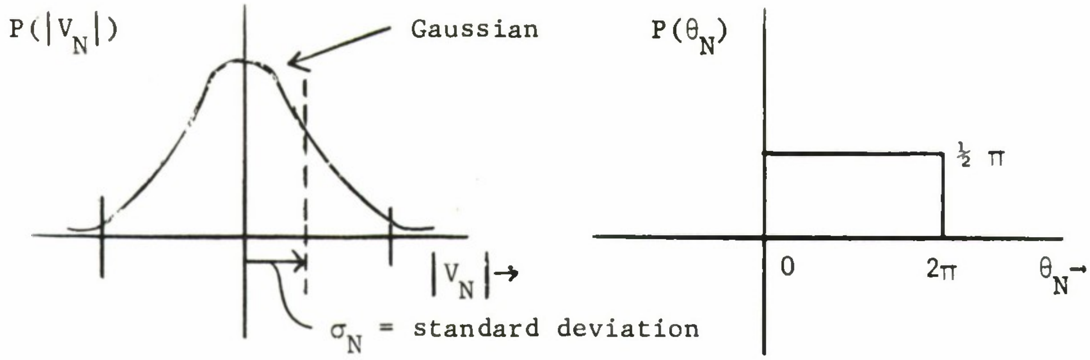
$$
R(\beta) = \begin{pmatrix} e^{-i\beta} & 0 \\ 0 & e^{i\beta} \end{pmatrix}
$$

3.3 Noise Generation and Normalization

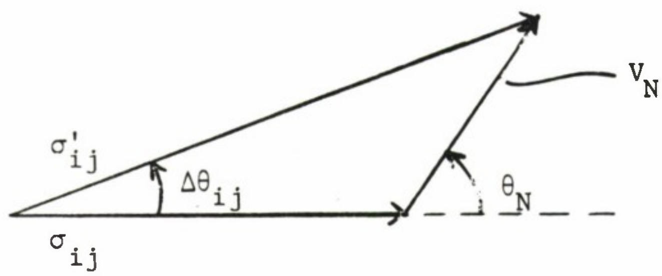
Using a random generator, independent uniformly distributed noise is obtained. This noise is then transformed into independent Gaussian noise (truncated) by a Gaussian transfer function. We have the random variable V_N such that

$$
V_N = |V_N| e^{i\theta_N}
$$

where the probability distributions are



These outputs are used to modify each of the entries of the scattering matrix, $\sigma_{i,j}$; $i,j = 1,2$ as follows.



$$|\sigma'_{ij}| = [(|\sigma_{ij}| + |V_N| \cos \theta_N)^2 + (|V_N| \sin \theta_N)^2]^{\frac{1}{2}}$$

$$\Delta \theta = \tan^{-1} \left(\frac{|V_N| \sin \theta_N}{|\sigma_{ij}| + |V_N| \cos \theta_N} \right)$$

If $V_1, V_2, \ldots, V_k, V_{k+1}, \ldots$ represents the sequence of outputs from the noise generator then for independent additive noise on the four entries of the scattering matrix we have

$$\text{at } t = t_k,\ \sigma_{11} + V_k,\ \sigma_{12} + V_{k+1},\ \sigma_{21} + V_{k+2},\ \sigma_{22} + V_{k+3}$$

$$t = t_{k+1} = t_k + \Delta t,\ \sigma_{11} + V_{k+4},\ \sigma_{12} + V_{k+5},\ \text{etc.}$$

Taking σ_N^2 = variance of $|V_N|$, the scattering data is normalized so that at maximum target cross-section σ_o (in square units) the stored signal value $e = 1$.

Now the received signal $E = \frac{k\sigma}{R^4}$, $\sigma = \sigma_o e$ where e is transmitted signal and R is the range. Thus,

$$\frac{S}{N} = \frac{k\sigma_o e}{KT(NF)R^4} = \frac{\gamma\sigma_o e}{R^4} = \frac{e}{N'},$$

where T is equvalent noise temperature, K is the Boltzman constant, NF is the system noise figure, and

$$N' = \frac{R^4}{\gamma\sigma_o} \quad \text{is noise power.}$$

Then the r.m.s. noise voltage $= (N')^{\frac{1}{2}}$ where

$$(N')^{\frac{1}{2}} = \frac{R^2}{(\gamma\sigma_o)^{\frac{1}{2}}} = \sigma_N$$

This provides the necessary scaling of noise level to e from E in terms of the range variation based on a system noise power of N.

The effect of noise then is to produce a scattering matrix.

$$S' = \begin{pmatrix} \sigma'_{11} & \sigma'_{12} \\ \sigma'_{21} & \sigma'_{22} \end{pmatrix} = \begin{pmatrix} |\sigma'_{11}|\, e^{j(\Delta\theta_{11} + \theta_{11})} & |\sigma'_{2}|\, e^{j(\Delta\theta_{12} + \theta_{12})} \\ |\sigma'_{21}|\, e^{j(\Delta\theta_{21} + \theta_{21})} & |\sigma'_{22}|\, e^{j(\Delta\theta_{22} + \theta_{22})} \end{pmatrix}$$

3.4 Polarization Modes and Radar Outputs

A general elliptically polarized field, E, can be written in terms of left circular, L, and right circular, R basis vectors as

† Further detail is given in internal reports noted on page 1.

$$E = A_L \, e^{j\phi_L} \, L + A_R \, e^{j\theta_R} \, R$$

Two parameters which distinguish the mode of transmission are ν , the ellipticity angle, and θ_p , the polarization angle, where

$$\tan \nu = \frac{A_R}{A_L}$$

$$\theta_p = \frac{\theta_R - \theta_L}{2}$$

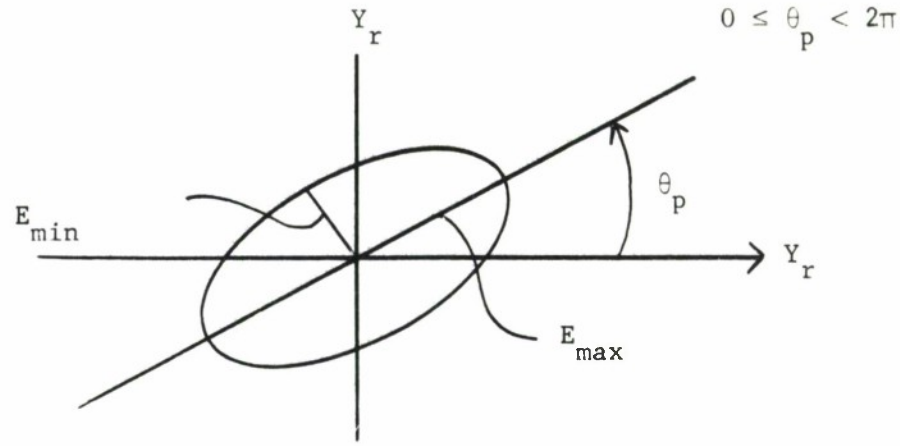
We see that for $0 < \nu < \pi/4$, $(A_L > A_R)$ giving left circulation for the field vector while for $\pi/4 < \nu < \pi/2$, $(A_R > A_L)$ giving right circulation. At the boundary $\nu = \pi/4$, $(A_R = A_L)$ linear polarization occurs.

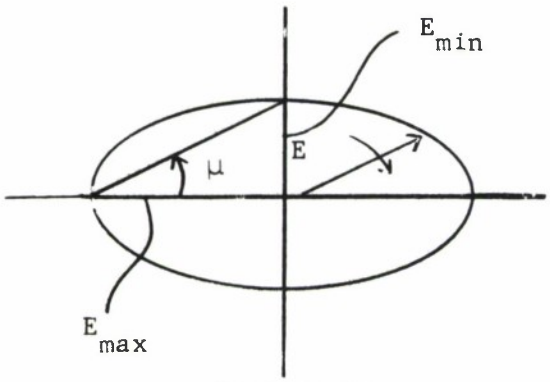
Indeed the mode of transmission or reception can be altered by a rotation of the antenna by θ_p and a change in the relative amounts of signal on the X_r, Y_r, radar axes by ν .

It can be shown that

$$\frac{|E_{max}|}{|E_{min}|} = |\cot \mu| ; \ \mu = \pi/4 - \nu$$

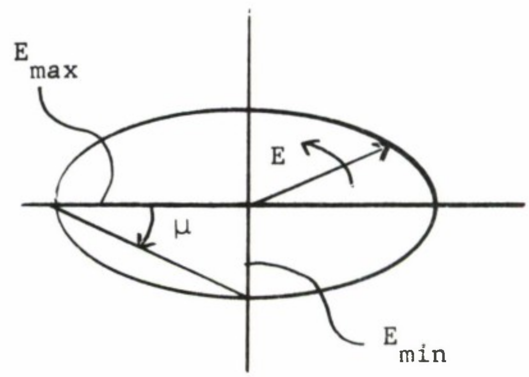
These results are shown in the following diagrams.





Left Circular

$\mu > 0$; $0 < \nu < \pi/4$



Right Circular

$\mu < 0$; $\pi/4 < \nu < \pi/2$

The line from which μ is referenced is drawn from E_{max} to the E_{min} reached by the field vector E. The counter clockwise and clockwise motions for right circular and left circular respectively then result in the two drawings.

The transmission and reception polarizations can be specified by the vectors

$$\vec{T} = \begin{pmatrix} e^{-j\theta_T} & \cos \nu_T \\ e^{j\theta_T} & \sin \nu_T \end{pmatrix} \quad \text{and} \quad \vec{R} = \begin{pmatrix} e^{-j\theta_R} & \cos \nu_R \\ e^{j\theta_R} & \sin \nu_R \end{pmatrix}$$

The radar output voltage is then given by

$$V_{RT} = \vec{R} \cdot S\vec{T}$$

with S given with respect to circular polarization.

The V_{RT} carry magnitude and phase information. For example, with left circular transmission and reception modes

$$\vec{T} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{R} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad V_{RT} = \sigma_{11}$$

Four such measurements are needed to obtain the four σ's of S. Using only magnitude measurements requires eight such measurements.

The θ_p can act in the same manner as antenna rotation (both transmission and reception involve θ_p) given by β_3. We let any rotation be β.

For example, with transmission and reception both left circular

$$\vec{T} = \begin{pmatrix} e^{-j\beta} \\ 0 \end{pmatrix}, \qquad \vec{R} = \begin{pmatrix} e^{-j\beta} \\ 0 \end{pmatrix}$$

then $V_{RT} = \sigma_{11} e^{-j2\beta}$.

The cross terms are invariant to β. For a cross term, we take $\vec{T}$ left circular and $\vec{R}$ right circular, that is

$$\vec{T} = \begin{pmatrix} e^{-i\beta} \\ 0 \end{pmatrix}, \qquad \vec{R} = \begin{pmatrix} 0 \\ e^{j\beta} \end{pmatrix}$$

then $V_{RT} = \sigma_{21}$.

To provide sufficient flexibility, eight possible $\vec{T}$ and eight possible $\vec{R}$ are allowed. This provides 64 combinations in establishing possible V_{RT} .

The $\vec{T}$ used are L, R, X, Y, 45° linear, 135° linear, and a general pair where ν and θ_p can be freely specified.

REFERENCES

1. J.F.A. Ormsby, "Motion Simulation of Ground Observed Earth Satellites," ESD-TR-69-185, The MITRE Corporation, July, 1969, originally issued as W-7289, The MITRE Corporation, 4 September 1964.

2. H.C. Thacker, Jr. and W.E. Milne, "Interpolation in Several Variables," SIAM Journal, Vol. 8, No. 1, March 1960.

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13. ABSTRACT

Using the outputs from the simulation of overall body motion related to observations at a complex of earth stations, this report develops a model for generating the scattering matrix and radar output voltages.

The general multistatic case is treated using a geometry which simplifies the data gathering and merges conveniently with the motion simulation. The monostatic case is given separate treatment due to certain simplifications possible in this situation.

Variations to account for polarization modes, Faraday rotation and noise are also included.

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